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An investigation of the role of spectroscopic factors in the breakup reaction of ^{11}Be*

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The experimental elastic cross section data of the projectile ^{11}Be on target ^{12}C at 49.3 MeV/nucleon energy is analysed. The calculations for the elastic scattering are performed by the phenomenological optical model. The different optical potentials to include breakup effects into the calculations, which are neutron+^{12}C, neutron+^{10}Be and ^{10}Be+^{12}C, are described with the aid of the global potentials for neutron interactions and fitted to experimental data for the core and target interaction. Also, the first analysis of the optical model for ^{10}Be on target ^{12}C at 39.1 MeV is done for building the interaction potential of the core and the target for ^{11}Be. For investigating the effects of the spectroscopic factors, obtained factors from the direct capture process using the nuclear level density are compared with the previous cross section and spectroscopic factor results. Obtained results for the elastic cross section reproduce the experimental data very well and show the requirement of including spectroscopic properties such as, spectroscopic factors and density of the excited states, to explain this elastic cross section data.

Keywords: Spectorscopic factors, Nuclear level density, Breakup reaction, Optical Model

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I. INTRODUCTION

Experiments with radioactive ion beams (RIBs) started a new era in nuclear reaction physics in the last decades [1–3]. In these experiments, which aim to probe and understand the nuclear structure, some unexpected properties of light exotic nuclei have been discovered. One of the most intriguing attributes is the halo structure [1], consisting of a core and weakly-bound valance nucleon(s). Up to now, this phenomenon has been greatly investigated experimentally on the various targets [4–6] and caused a challenge for nuclear reaction theoreticians to reproduce the experimental data [4, 7, 8].

^{11}Be is one of the four one-neutron halo nuclei together with ^{19}C [9], and newly reported ^{31}Ne [10] and ^{37}Mg [11]. Some experiments have been conducted for understanding the structure of ^{11}Be. Firstly, Tanihata et al. [3] observed a large radii for ^{11}Be, compared to ^{10}Be, in cross section measurements with targets at 790A MeV and found the halo structure for ^{11}Be originating from its small neutron separation energy of 0.503 MeV. A few years later, Fukuda *et al.* [12] confirmed this conclusion in elastic scattering of ^{11}Be on C and Al targets at 33 MeV/nucleon. Since these distinguished works, ^{11}Be has been continuously studied experimentally [6, 13–16] and theoretically [17–19].

One experimental study of ^{11}Be is performed by Cortina-Gil *et al.* [6] for the cross section of the elastic scattering on ^{12}C at 49.3 MeV/nucleon incident energy. The first theoretical investigation of this measurement is an adiabatic approximation, assuming no internal motion between the valance nucleon and the core in projectile, and also neglecting the interactions between the valance nucleon and the target nucleus [8]. Also, in the same year, Al-Khalili *et al.* [20] investigated the same reaction with the few-body Glauber model, in which the particles of the projectile are considered as following straight line paths through the interaction field of the target. In addition to these studies, the continuum-discretized coupled-channels (CDCC) method was applied to this elastic scattering by Takashina *et al.* [21], and also they used the same parameter set for the optical potentials between the projectile components and the target as in Ref. [20]. In this non-adiabatic method, due to the very low neutron or proton separation energy, the continuum states of the projectile above this threshold energy are discretized to a finite number of states using momentum bins. Including the breakup effects into the theoretical calculations of the mentioned methods gives almost the same results.

In the present study, the elastic scattering of the projectile halo nucleus ^{11}Be on the target ^{12}C at 49.3 MeV/nucleon [6] is investigated as a breakup reaction using the optical model with the aid of a nuclear structure model. Different from the other studies, the optical model potential used for the interaction between the core nucleus ^{10}Be and the target ^{12}C is obtained by fitting to elastic cross section data at 39.1 MeV/nucleon. This data [13] is investigated with the optical model for the first time in this study. As for the interaction between the valance neutron of the halo nucleus and the target, the optical potential is deduced from an interpolation for different incident energies of neutron on ^{12}C target by means of the global potential of Ref. [22]. In order to describe non-elastic contributions, we use a surface potential, named as DPP (dynamical polarization potential) or VCP (virtual coupling potential), in our calculations. Finally, a binding potential is employed for the n+^{10}Be system. Unlike similar studies, we determined the value of the spectroscopic factor, describing the wave function of ^{11}Be in terms of the wave function of ^{10}Be, with the method given in Ref. [23] for the direct neutron capture reaction ^{10}Be+n$\rightarrow$ ^{11}Be+γ. However, we used a new nuclear level density (NLD) model [24], which strongly depends on the deformation of the nucleus.

This paper is organized as follows: The method used in this study is presented in Section II, the results obtained by

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this method are given for the breakup reaction of ^{11}Be in Section III, and finally in Section IV, concluding remarks drawn from this study are given.

II. THEORY

Since the mid-fifties, the optical model has been widely used to investigate the elastic scattering cross section for both light and heavy ions in a wide range of incident energies. The optical model considers the projectile and the target nuclei as structureless particles in order to avoid many-body problems in nuclear physics calculations, and describes the interaction between the projectile and target with an effective potential. In this work, since we included breakup effects, the halo projectile ^{11}Be is considered as a two-body system, which consists of a ^{10}Be core and a valance neutron. Therefore, we define effective potentials between projectile components and the target ^{12}C, which are n+^{12}C, ^{10}Be+^{12}C, and n+^{10}Be, as used in Ref. [25]

$$U_{\text{eff}} = U_{\text{CT}} + U_{\text{VT}} + U_{\text{CV}}, \tag{1}$$

where C, T, V correspond to the ^{10}Be core, the ^{12}C target and the valance nucleon, respectively. An effective potential is a combination of the following terms as

$$U(r) = V_l(r) + V_{\text{C}}(r) + V_{\text{Vol}}(r) + V_{\text{Sur}}(r) + V_{\text{SO}}(r). \tag{2}$$

The first term is the centrifugal potential, which is traditionally defined as

$$V_l(r) = \frac{\hbar l(l+1)}{2\text{m}r^2}. \tag{3}$$

Uniformly charged sphere assumption is employed for the nucleus

$$V_C(r) = \begin{cases} \dfrac{Z_P Z_T \text{e}^2}{2R_C}\left(3 - \dfrac{r^2}{{R_C}^2}\right) & r \le R_C \\ \dfrac{Z_P Z_T \text{e}^2}{r} & r \ge R_C \end{cases}, \tag{4}$$

where the charge radius R_c is defined as $R_c = r_c(A_P^{1/3} + A_T^{1/3})$, the Coulomb potential parameter r_c is taken as 1.2 fm in this work. In the optical model, the volume term in an effective potential has a crucial role and can be described with the real part of this term. However, for inelastic contributions, an imaginary part is added to the volume term for the purpose of considering absorption of the reaction flux from the elastic channel to the inelastic reaction channels. Therefore, conventionally the volume term consists of real and imaginary parts in the reaction studies

$$V_{\text{Vol}}(r) = \frac{-V_0}{1 + \exp\left(\frac{r - R_{\text{v}}}{a_{\text{v}}}\right)} + \frac{-iW_0}{1 + \exp\left(\frac{r - R_{\text{w}}}{a_{\text{w}}}\right)}, \tag{5}$$

where potential depths, radii, and surface diffuseness parameters for both real and imaginary parts should be adjusted to fit elastic scattering data. Even if the investigated reaction is the elastic scattering, non-elastic contributions can still exist in the elastic channels. To include these contributions, the surface potential is used

$$V_{\text{Sur}}(r) = \frac{-4V_0 \exp\left(\frac{r - R_{\text{v}}}{a_{\text{v}}}\right)}{\left[1 + \exp\left(\frac{r - R_{\text{v}}}{a_{\text{v}}}\right)\right]^2} + \frac{-4iW_0 \exp\left(\frac{r - R_{\text{w}}}{a_{\text{w}}}\right)}{\left[1 + \exp\left(\frac{r - R_{\text{w}}}{a_{\text{w}}}\right)\right]^2}, \tag{6}$$

which is in derivative form of the volume term. The final term in Eq. (2) is the spin-orbit (SO) potential

$$V_{\text{SO}}(r) = \left(\frac{\hbar}{m_\pi c}\right)^2 \frac{1}{r}\frac{\text{d}}{\text{d}r} \frac{V_{\text{SO}}}{\left[1 + \exp\left(\frac{r - R_{\text{SO}}}{a_{\text{SO}}}\right)\right]} 2L \cdot s, \tag{7}$$

where $(\hbar/m_\pi c)^2 = 2\,\text{fm}^2$.

The optical potential parameters in these equations can be determined from elastic scattering data. As a first step in fitting procedure of potential parameters, the geometrical parameters are adjusted to positions of peaks occurred in data. Afterwards, the potential depths of all used optical model potentials are fitted to experimental data to give the minimum χ^2 value.

In the case of the halo nucleus ^{11}Be, the spectroscopic factor as a structure property is used to describe the ground state and the first excited state of ^{11}Be in terms of ^{10}Be. The spectroscopic factor can be determined from the fitting to experimental cross section data of transfer or direct capture processes, and also they can be obtained theoretically from the shell model calculations. In the literature, many transfer processes include the spectroscopic factor value of ^{11}Be for ^{9}Be(t,p)^{11}Be [26–29], ^{10}Be(d,p)^{11}Be [30–33] and ^{11}Be(p,d)^{10}Be [34] reactions. However, the experimental data of the direct capture cross section for ^{10}Be(n,γ)^{11}Be is not available, but the direct capture cross section data can be deduced from Coulomb dissociation [35]. As a tool for calculations of the light ion cross sections, such as direct capture processes, the nuclear level density has a crucial role of reproducing the measured data and defining the spectroscopic factor. Therefore, the relation between the direct capture cross section and the nuclear level density, which is the number of the excited levels around an excitation energy, can be defined as [23]

$$\sigma^{\text{DC}}(E) = < S > \int_0^{Sn} \sum_{J_f, \Pi_f} \rho(E_f, J_f, \Pi_f) \sigma_f^{\text{cont}}(E) dE_f, \tag{8}$$

where S represents the average spectroscopic factor and ρ is the level density function in terms of the excitation energy E_f, total angular momentum J_f, and the parity Π_f of the compound nucleus. In the present work, we calculate the direct neutron capture cross section and compare to deduced data [35] from Coulomb dissociation of ^{11}Be measured by Nakamura *et al.* [36]. To do this calculation, a Laplace-like formula [24] is used for the energy dependence of the nuclear level density parameter in the Fermi gas model. According to this formula, the level density parameter strongly depends on the deformation of the nucleus, and the results obtained with

this formula are very successful to describe low-lying collective levels compared to other phenomenological level density models [37]. Therefore, keeping in mind that ^{10}Be and ^{11}Be are well-deformed nuclei, we expect that this formula is convenient to explain the neutron capture cross section data of ^{10}Be. In the following section, we will give the optical potential parameters which are used in this study and the results of our calculations.

III. RESULTS AND DISCUSSION

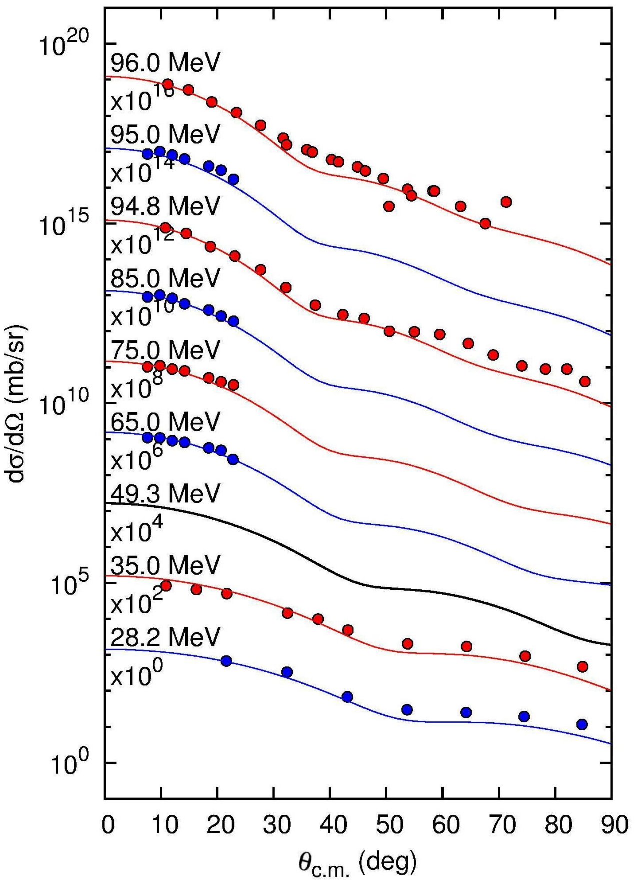

Fig. 1. (Color online) Cross sections for n+^{12}C at 28.2 MeV, 35.0 MeV [38], 65.0 MeV [39], 75.0 MeV [39], 85.0 MeV [39], 94.8 MeV [40], 95.0 MeV [39], 96.0 MeV [41]. Obtained results using the optical potentials for 49.3 MeV incident energy are represented by black line.

To describe the interactions between the projectile and the target, we consider the weakly-bound nucleus ^{11}Be as ^{10}Be+n. For this purpose, first we focus on the interaction between the neutron and the target. A great number of experimental data in 0–100 MeV energy range [38–41] is found for the elastic scattering of the neutron on ^{12}C and can be used to define the effective potential between the valance nucleon and the target in this case. Unfortunately, for 49.3 MeV incident energy, no experimental data is available. Thus, an interpolation of the global parametrization [22] is used. The results obtained with this global potential are given in Fig. 1. As seen from figure, this interpolation of the global parametrization for n+^{12}C at 49.3 MeV incident energy is in good agreement with a wide range of energy.

Table 1. Adjusted potential parameters for n+^{12}C, n+^{10}Be, ^{10}Be+^{12}C and ^{11}Be+^{12}C interactions. r_c is taken as 1.20 fm for the Coulomb interaction.

Interaction Potential	Type	V_0(MeV) W_0(MeV)	r_v(fm) r_w(fm)	a_v(fm) a_w(fm)
n+^{12}C	Volume	37.5	1.127	0.676
		4.90	1.127	0.676
	Surface	0.00	1.306	0.543
		4.15	1.306	0.543
	Spin-Orbit	4.68	0.903	0.590
		–0.39	0.903	0.590
^{10}Be+^{12}C	Volume	15.049	0.950	0.580
		23.326	1.100	0.630
n+^{10}Be	Volume	37.5	1.127	0.676
^{11}Be+^{12}C	Surface	42.793	0.950	0.580
SF=1.48		3.935	1.100	0.530
^{11}Be+^{12}C	Surface	29.635	1.100	0.580
SF=0.71,0.62		1.036	1.100	0.530

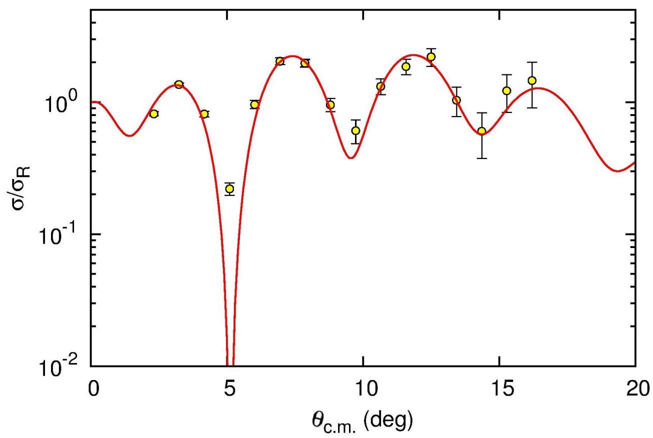

Fig. 2. (Color online) Cross sections for ^{10}Be+^{12}C target at 39.1 MeV. Solid red line represents the results obtained by using the optical potential parameters given in Table 1. The experimental data is taken from [13].

In contrast to ^{11}Be, very long-lived ($T_{1/2} = 1.5 \times 10^6$ y) and a tightly-bound nucleus ^{10}Be has a greater neutron separation energy of 6.81 MeV. One experimental study about ^{10}Be is Lapoux *et al.* [13], in which they measured the elastic cross section for ^{10}Be and ^{11}Be projectiles on proton and ^{12}C targets at 39.1 MeV/nucleon and 38.4 MeV/nucleon, respectively, and this data was investigated using the microscopic Jeukenne-Lejeune-Mahaux nucleon-nucleus potential for the proton target and the folding model for the C target. Unlike the other studies [20, 21], in order to be more physical and reliable, the potential parameters describing the interaction between the core and the target are adjusted to the elastic scatter-

ing data at 39.1 MeV/nucleon energy [13]. Our obtained values of the potential depth parameters are shallow compared to their potential. We use the experimental β_2 quadrupole deformation value, which is –0.6 [42], for the first (2^+) excited level of ^{12}C, which is 4.4 MeV. Also, in order to take into account the non-elastic contributions caused by the interactions at the surface region, additionally one can add the surface term to the effective potential. This potential is sometimes referred as a surface term or derivative form of Woods-Saxon potential or DPP or VCP, and can be obtained by different methods. The parameters of DPP can be obtained from microscopical [43–45] or phenomenological [46–52] calculations by fitting to the experimental data. For ^{10}Be+^{12}C, we used a phenomenological DPP obtained from the fit process to the experimental data combined with a volume term. Obtained results for this elastic scattering and the optical potential parameters used in this calculation are given in Fig. 2 and Table 1, respectively. With the exception of the well-known phenomenon at 5°, the data is reproduced well.

Many authors analysed the elastic scattering of the halo-nuclues ^{11}Be on target ^{12}C for this incident energy by different theoretical models [8, 20, 21]. However, none of these studies are incorporated the nuclear structure to explain the data. On the other hand, adding the nuclear structural information into the reaction calculations for such a weakly-bound system as the halo nucleus played a crucial role in contributing to the agreement between the predictions and the experimental data. The spectroscopic factor as a nuclear structure property is one of the most important ingredients for the theoretical cross section calculations of both light and heavy ions. There are many methods which can be used for determining the value of the spectroscopic factor. Of course, the easiest method is to fit the spectroscopic factor values to the experimental cross section data directly, but the traditional way of estimating the spectroscopic factor is to use the shell model, in which the spectroscopic factor is defined as the square of normalization of the overlap integral between the wave function of the valance nucleon in the state of the target nucleus and the residual nucleus. Also, the spectroscopic factor is a key ingredient for the direct capture process for which the related cross section often dominates the total cross section at the very low energies of astrophysical interest. The direct capture process can be used for obtaining the spectroscopic factor and is known to play a notable role in light exotic nuclei systems for which few, or even no resonant states are available. Although many works containing the spectroscopic factors derived from the transfer processes exist for the halo nucleus ^{11}Be, the direct neutron capture cross section data for ^{10}Be to compose ^{11}Be is not available in the literature. However, the direct capture cross sections can be obtained from the data of the Coulomb dissociation.

In obtaining the spectroscopic factor with the aid of the direct capture cross section calculations, the most important component is the nuclear level density. Generally, the reasons for not trusting level density models in such calculations are their insufficient agreements with the experimental observables and their way of taking into account the collective effects. For overcoming these challenges, recently, we introduced a new Laplace-like formula [24] for the NLD parameter to improve the predictive power for describing the low-lying collective levels, which are well known to be of vital importance for the direct capture process. With this formula, good agreement is achieved with the experimental observables. Therefore, the direct neutron capture cross section calculation based on this level density model for the ^{10}Be(n,γ)^{11}Be processes is shown in Fig. 3. Although the data could not be reproduced below 0.5 MeV, in the rest of the energy range the same behaviour is well explained. The average value for the spectroscopic factor is determined as 1.48 from the least chi-square fit. The value of the parameters used in our level density calculation are 1.345 for the asymptotic level density parameter $\tilde{a}$ and 0.285 for the deformation parameter β obtained from the fit to discrete levels of ^{11}Be, which were taken from our previous study [24].

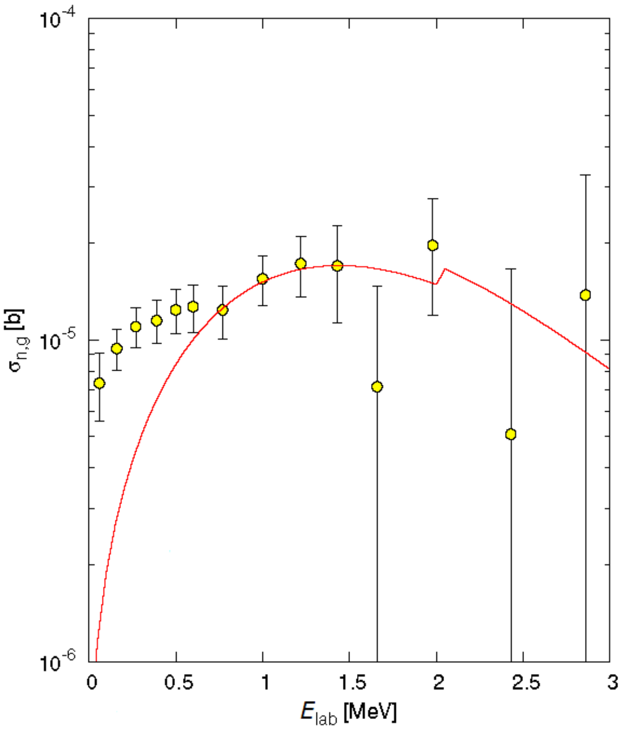

Fig. 3. (Color online) The direct neutron capture cross section results for ^{10}Be(n+γ)^{11}Be reaction at 0–3 MeV lab. energy. The solid red line represents the results of the present work using the level density model [24], and obtained spectroscopic factor value is 1.48. The deduced experimental data from Coulomb dissociation data of ^{11}Be [36] is taken from Ref. [35].

Considering ^{11}Be as a two-body projectile, all values of the optical potential parameters are given in Table 1 for the n+^{12}C, n+^{10}Be and ^{10}Be+^{12}C interactions. The parameter values of potential depths for a ^{10}Be+^{12}C at 39.1 MeV incident energy are rearranged as 46.3 MeV and 13.8 MeV of real and imaginary parts, respectively. The same procedure is repeated for the surface potential as 9.820 MeV and 3.661 MeV. Also, to include the non-elastic contributions of ^{11}Be+^{12}C, a surface potential is added to effective potential. Moreover, to com-

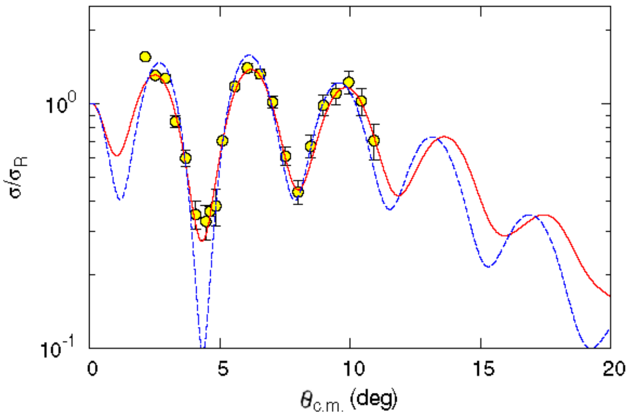

Fig. 4. (Color online) The elastic cross section results for ^{11}Be+^{12}C at 43.9 MeV. The dashed blue line is the breakup calculation with the spectroscopic factors 0.71 for the ground state and 0.62 for the first excited state [53]. The solid red line represents the cross section result with the spectroscopic factor value of 1.48 obtained from the direct capture cross section. The experimental data is taken from [6].

pare our results, we performed another calculation with the spectroscopic factor obtained through the transfer reactions by Schmitt *et al.* [53], which is 0.71 for the ground state and 0.62 for the first excited state, respectively. The results of this calculation are also shown in Fig. 4 with dashed blue line. In our calculations the average value of the spectroscopic factor is taken as the spectroscopic factor of the ground state. Since the spectroscopic factor of the first excited state has less effect on the results, the value of this factor is taken as 1.0. Finally, our prediction for the elastic scattering cross section of ^{11}Be on ^{12}C is shown in Fig. 4 with a solid red line. The inclusion of the nuclear level density with the Laplace-like formula in the reaction calculations has a positive effect on reproducing the cross section data. Also, the fit method we used for the optical potential parameters, which is to adjust the geometrical parameters to positions of peaks and the depths to give a minimum χ^2, effected the agreement in a positive way.

IV. CONCLUSIONS

In summary, we have investigated the elastic scattering cross section data of the projectile ^{11}Be on a ^{12}C target at 49.3 MeV/nucleon [6]. To include breakup effects into the calculations, the different optical potentials for n+^{12}C, n+^{10}Be and ^{10}Be+^{12}C are described. Also, the present study contains the first analysis of the phenomenological optical model for 39.1 MeV incident energy of the projectile ^{10}Be on a ^{12}C target. The obtained results are in better agreement with the experimental data, compared to the microscopic study of Lapoux *et al.* [13], which is the first and the only study of this reaction.

Not only the effects of including the spectroscopic factor into the calculations were found to be significant for the breakup reaction of ^{11}Be, but also, adjusting the geometrical parameters to positions of peaks and the depths to give minimum χ^2, positivly contributes to reproducing the scattering data.

The theoretical framework used for obtaining the spectroscopic factor by using the nuclear nuclear level density to calculate the direct neutron capture cross section is employed for the first time in the breakup reaction calculation of ^{11}Be. Moreover, the nuclear level density is used for the first time as a spectroscopic tool in a light exotic nuclei induced reaction. Consequently, besides the success of the nuclear level density with the Laplace-like formula for the level density parameter [24] as a structure model, the results show that this new method seems appropriate to perform the reaction calculations.

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